Comment on "Geometric Dilution of Precision in Global Positioning System Navigation"

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FANG[1] has given an algorithm to compute the Geometric Dilution of Precision (GDOP) required in Global Positioning System Navigation. This algorithm basically makes use of matrix inversion algorithm in Ref. 2 to compute GDOP. It is not necessary to compute the inverse of a matrix to determine GDOP. This can be shown as follows with the help of Cayley-Hamilton Theorem[3]:

$$\det(sI - P) = \sum_{i=0}^{4} p_{4-i} s^i \qquad (p_0 = 1) \qquad (1)$$

and

$$\det(sI - P^{-1}) = \sum_{i=0}^{4} \frac{p_i}{p_4} s^i \qquad (p_4 \neq 0) \qquad (2)$$

where $P = H^T H$, a 4×4 position-time navigation error covariance matrix, H is a 4×4 psuedo range measurement matrix, I is a 4×4 identity matrix, s is a complex variable, and p_i's are coefficients of characteristic equation of P. "det" and superscript T denote the determinant and transpose of the matrix respectively.

From Eqs. (1) and (2)

$$GDOP = \text{trace } P^{-1} = p_3/p_4 \qquad (3)$$

Thus GDOP can be obained from p_3 and p_4 *without* computing the inverse of P.

References

[1] Fang, B.T., "Geometric Dilution of Precision in Global Positioning System Navigation," *Journal of Guidance and Control,* Vol. 4, Jan.-Feb. 1981, pp. 92-94.

[2] Householder, A.A., *Principles of Numerical Analysis,* McGraw Hill, New York, 1953.

[3] Bellman, R., *Introduction to Matrix Analysis,* McGraw Hill, New York, 1960.

Reply by Author to P. S. Kamat

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KAMAT's Comment concerning a minor part of the author's Engineering Note is misdirected. The algorithm given by the author for the computation of GDOP is meant for use in the selection an optimum GPS configuration among many possible ones. An indiscriminate use of Kamat's Eq. (3) for this situation would be much less efficient than the author's Eq. (10). It should also be pointed out that Kamat's Eq. (3) is equivalent to the Cramer's rule in computing the diagonal elements of the matrix inverse, and may not even be the most efficient in the computation of a single GDOP. In the preparation of his Comment, the very least he could have done was to compare the operations counts of his algorithm versus other efficient methods.

Errata

Space Guidance Evolution— A Personal Narrative

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[J. Guidance, **5**, 97, (1982)]

TWO key references were omitted from this article after the galley proofs had been corrected. They are:

[19] Battin, R.H., "Computational Procedures for the Navigational Fix," Appendix B of "Interplanetary Navigation System Study," Report R-273, MIT Instrumentation Laboratory, Cambridge, Mass., April 1960.

[20] Kalman, R.E., "A New Approach to Linear Filtering and Prediction Problems," *Journal of Basic Engineering, Transactions of the American Society of Mechanical Engineers,* Vol. 82D, March 1960, pp. 35-45.

Also, for the record, two small typographical errors should be reported: 1) the lower case f in Eq. (2) should be an upper case F; 2) the third plus sign in Eq. (12) should be omitted; i.e., the last term in the equation is $\frac{1}{2} g t_{ff}^2$.